



Teaching for Mastery Calculation Policy

The National Curriculum 2014

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject.

The aim of this policy

Mastery is for all, and the aim of this policy is to ensure consistent strategies, models and images are used across our school to embed and deepen children's learning and understanding of mathematical concepts. It has been designed to support the planning and teaching of mathematics in our school and should be used in conjunction with our school mathematics curriculum documents.

Calculation strategies

This policy details the progression in strategies to use when calculating, therefore teachers must plan opportunities for pupils to apply them within mathematics lessons and in other opportunities across our curriculum. It details the progression of strategies and written methods which children will be taught as they develop their understanding of the four operations. Strategies are set out in a Concrete, Pictorial, Abstract (CPA) approach to develop children's understanding and mastery of mathematical concepts as they learn how to use these methods.

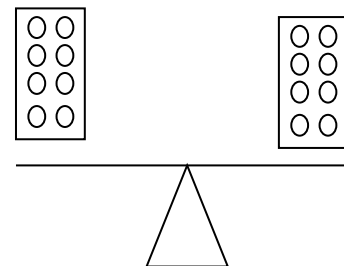
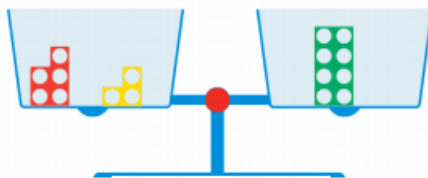
Children use concrete objects initially to help them make sense of the concept or problem; this could be real objects such as fruit or toy bears, or more abstract concrete representations such as cubes or counters. This is then developed through the use of images, models and children's own pictorial representations before moving on to abstract mathematics. Children will travel through this approach again and again, often revisiting previous stages when a concept is extended. Concrete, pictorial and abstract models can also be used alongside each other to deepen understanding and support 'proving' and 'working out'. They are designed to equip children with a 'tool box' of skills and strategies that they can apply to solve problems in a range of contexts. As children become increasingly independent, they will begin to develop an understanding of when to use the strategies.

The strategies are separated into the 4 operations for ease of reference. However, teaching addition and subtraction, and multiplication and division together supports children in making connections and seeing relationships.

Effective teaching of the strategies relies on increasing levels of number sense, fluency and being able to reason mathematically. Children must be supported to gain depth of understanding within the strategy through the CPA approach, and not learn strategies as a procedure.

Teaching equality

It is important that when teaching the 4 operations that equality (=) is also taught appropriately. Misconceptions that = means that children must 'do something' and that it indicates that an answer is needed can be common and should be addressed early on. Teachers should present children with number sentences and problems which place the = sign in different positions, different contexts and include missing number problems. For example, $?+4=7$; $7=3+?$; $<$, $>$, or $= 5+6$ ___ $7+4$. Balance scales and Numicon provide a useful resource to demonstrate equality as do pictorial representations. For example:



The importance of vocabulary

The 2014 National Curriculum places great emphasis on the importance of pupils using the correct mathematical language as a central part of their learning. Children will be able to articulate their mathematical reasoning more clearly and concisely if they use the appropriate mathematical vocabulary. Mathematical misconceptions, either in current or later learning, can also be avoided through the use of the correct vocabulary. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary by adults so that children use it too. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. Current mathematical vocabulary being used in lessons should be visible on the classroom working wall.

High expectations of the mathematical language used are essential, with teachers modelling and only accepting what is correct. For example:

✓	X
ones zero number sentences / expressions / equations / calculations digit face vertex / vertices regrouping / exchanging sum / difference / product / quotient	units 'oh' sums (unless for addition totals) number side corners borrowing answer

Pupils' opportunities to use mathematical vocabulary should be enhanced by teachers through the use of open questioning and collaborative learning with a partner or small group.

The use of the following precise mathematical vocabulary should be introduced to support children's understanding of written methods, in a similar way to introducing precise phonetics vocabulary such as phoneme and split digraph. It can be used effectively to discuss missing number problems and to prepare children for Key Stage 2 mathematics.

Addition (from FS2) <i>part + part = whole, whole = part + part, progressing to multiple parts</i> part – a portion of the complete amount whole – the complete amount Addition (from Y1) <i>addend + addend = sum, sum = addend + addend</i> addend – the amounts that you add sum – the result of the addition	Subtraction (from spring Y1) <i>minuend – subtrahend = difference, difference = minuend – subtrahend</i> minuend – the amount you start with and decrease/diminish subtrahend – the amount that you subtract difference – the result of the subtraction
Multiplication (from Y2) <i>multiplicand x multiplier = product, product = multiplicand x multiplier</i> multiplicand – the amount being increased or multiplied multiplier – the number by which another number is multiplied product – the result of the multiplication	Division (from Y2) <i>dividend ÷ divisor = quotient, quotient = dividend ÷ divisor</i> dividend – the amount to be divided into groups divisor – the number by which another number is divided quotient – the result of the division

End of year expectations for calculations

EYFS	Year 1	Year 2
Number: - have a deep understanding of number to 10, including the composition of each number - subitise up to 5 - automatically recall number bonds up to 5 (including subtraction facts) and some number bonds up to 10, including doubling facts Numerical Patterns: - verbally count beyond 20, recognising the pattern of the counting system - Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity - explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs - represent and use number bonds and related subtraction facts within 20 - add and subtract one-digit and two-digit numbers to 20, including zero - solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = ? - 9$ - solve one-step problems involving multiplication and division, by calculating the answer using concrete objects - solve one-step problems involving multiplication and division using pictorial representations and arrays with the support of a teacher	- apply an increasing knowledge of mental and written methods recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100 - add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers and adding three one-digit numbers - show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot - solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures - recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems - calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equals (=) signs - recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers to 100 - show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot - solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Progression in Calculations – ADDITION

Nursery and Foundation Stage 2

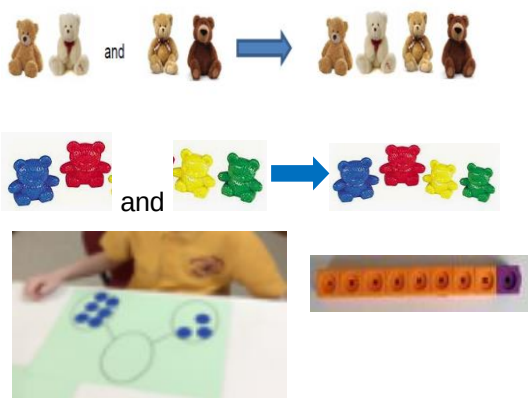
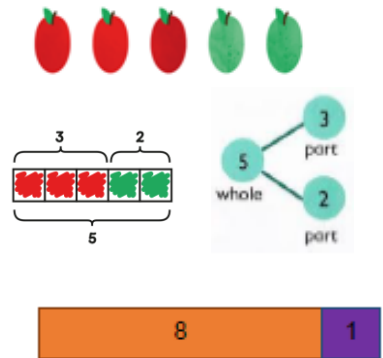
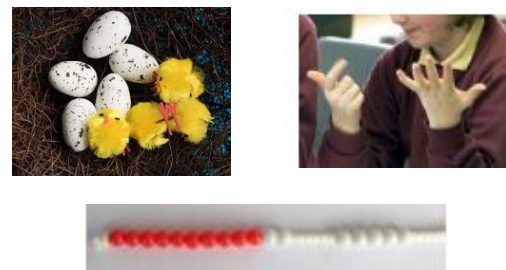
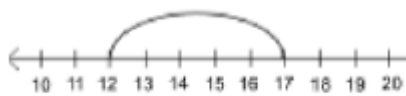
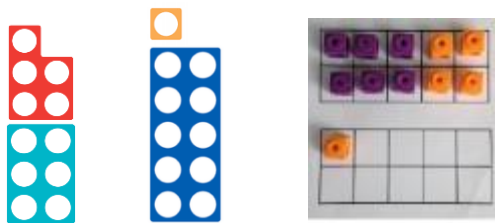
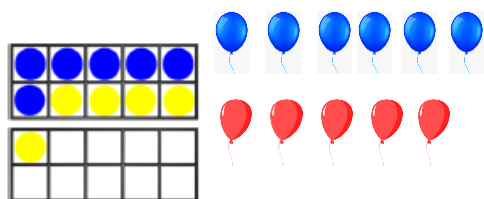
Before addition is introduced, children should have a secure knowledge of number.

In our Nursery, children are introduced to the concept of counting forwards, number order and number recognition through practical activities, counting songs, games, stories and real-life examples. Children learn how to count using one-to-one correspondence (pointing to each object as they count) and learn that anything can be counted, for example, claps, steps and jumps as well as objects. Children have opportunities to apply their knowledge of number and counting skills in indoor and outdoor provision.

In Foundation Stage 2 children develop a more secure knowledge of counting forwards, interpreting representations and comparison using 'more', before progressing to the composition of numbers and addition of two parts.

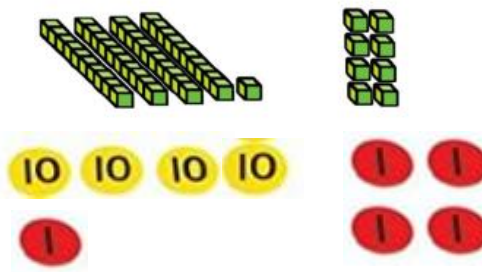
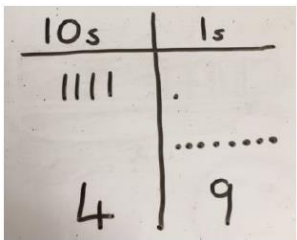
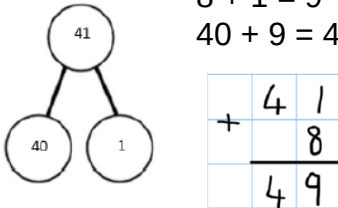
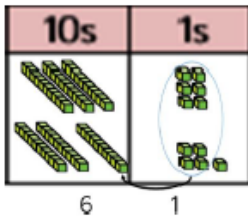
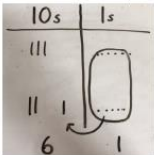
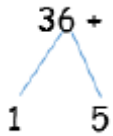
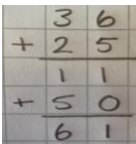
Progression in Calculations – ADDITION

Key vocabulary: part, whole, add, plus, altogether, total, more, is equal to, is the same as, addend, sum

Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole Count sets of objects, combine them and then re-count using 1:1 correspondence.	 Concrete strategy: teddy bears and interlocking blocks.	 Pictorial strategy: apples, ten frames, and number bonds.	$3 + 2 = ?$ I have 3 apples and I pick 2 more, how many do I have altogether? $3 + 2 = 5$ I have 3 apples and I pick 2 more, now I have 5 altogether. 3 is a part, 2 is a part and the whole is 5
Counting on Use objects, Numicon, bead strings and number lines starting with the biggest number and counting on.	 Concrete strategy: eggs, fingers, and bead strings.	Number lines can be used to encourage counting on rather than counting all. 	$5 + 12 = ?$ Reinforce counting on from the largest number. $7 + 3 = ?$ Encourage recall of known number facts.
Regrouping to make 10 Use tens frames and counters/cubes or Numicon to start using number bond knowledge to make and bridge 10.	 Concrete strategy: Numicon blocks and a tens frame.	Children draw the ten frame and counters or own picture. 	$6p + 5p = ?$ I have 6p, how much more do I need to make 10p? How much more do I add on now?

Progression in Calculations – ADDITION

Key vocabulary: part, whole, add, plus, altogether, total, more, is equal to, is the same as, addend, sum

Strategies	Concrete	Pictorial	Abstract
Adding 3 single digits Use as an opportunity to develop fluent recall and application of known number facts including bonds and doubles. Follow on from making 10, make 10 with two digits first if possible.		 $\textcircled{6} + 3 + \textcircled{4} = 13$	$4 + 7 + 6 = ?$ $4 + 6 + 7 = 10 + 7$ $= 17$ I know that 4 add 6 is 10 then I add 7.
Partitioning to add a two digit number and ones (TO + O) Use to develop a deep understanding of place value. <i>Add ones first to prepare for column addition.</i>	Use base 10 and PV counters 	Children represent base 10 with lines for tens and dots/circles/squares for ones. 	$41 + 8$ $8 + 1 = 9$ $40 + 9 = 49$  1 add 8 is 9, 40 add 9 is 49 so 41 add 8 is 49.
Partitioning to add 2 two digit numbers (TO + TO) Use to further develop understanding of place value and partitioning. <i>Add ones first to prepare for column addition.</i>	Exchange ten ones for one ten. 	Children represent base 10 and show exchanging on a place value chart. Add ones then tens to prepare for formal methods. 	Looking for ways to make 10 $36 + 25 =$  $30 + 20 = 50$ $5 + 5 = 10$ $50 + 10 + 1 = 61$  Add ones then tens.

Progression in Calculations – SUBTRACTION

Nursery and Foundation Stage 2

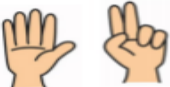
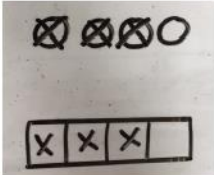
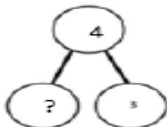
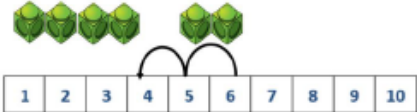
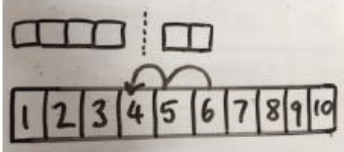
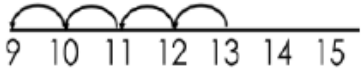
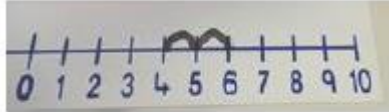
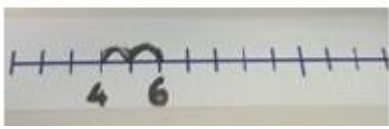
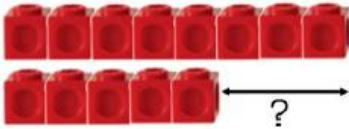
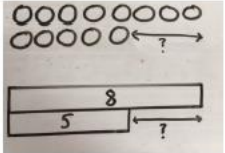
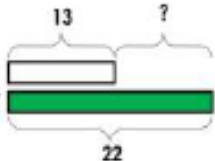
Before subtraction is introduced, children should have a secure knowledge of number.

In our Nursery, children are introduced to the concept of counting backwards, number order and number recognition through practical activities, counting songs, games, stories and real-life examples. Children learn how to count using one-to-one correspondence (pointing to each object as they count) and learn that anything can be counted, for example, claps, steps and jumps as well as objects. Children have opportunities to apply their knowledge of number and counting skills in indoor and outdoor provision.

In Foundation Stage 2 children develop a more secure knowledge of counting backwards, representations and comparison using 'less', before progressing to the composition of numbers and subtraction of a part.

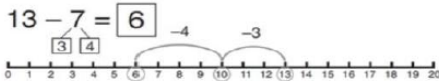
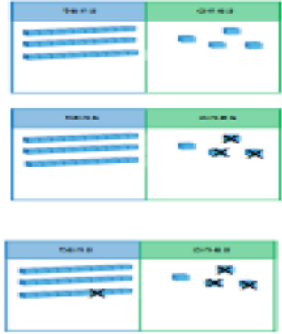
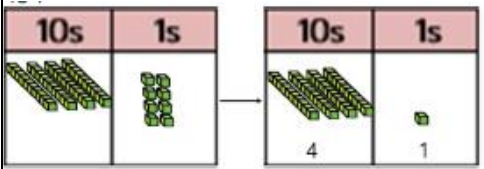
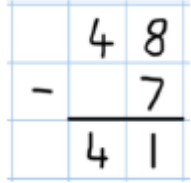
Progression in Calculations – SUBTRACTION

Key vocabulary: less than, fewer, take away, subtract, minus, minuend, subtrahend, difference

Strategies	Concrete	Pictorial	Abstract
Taking away objects and removing from a whole Use toys, bean bags, cubes, counters to physically take away objects, before taking away by crossing out representations.	 "10 take away 5 leaves 5"  $5 - 3 = 2$ 	 $5 - 1 = 4$ 	$8 - 2 = ?$ There are 8 cakes in a shop. Two cakes are sold. How many are left?   
Counting back Use objects, Numicon and number lines to subtract small numbers from larger ones.	 $6 - 2 = 4$ 	Children draw what they see.  	  Put 6 in your head, count back 2. What number are you at?
Finding the difference Develop a good understanding of 'the difference' between the largest and smallest number. Use to demonstrate the inverse relationship with addition.	Use practical resources to see 'the difference'. 	Children draw what they have used or draw a bar model.  	Find the difference between 8 and 5 $8 - 5 = \square$, the difference is ? Explore why $9 - 6 =$, $8 - 5 =$, $7 - 4$ have the same difference.

Progression in Calculations – SUBTRACTION

Key vocabulary: less than, fewer, take away, subtract, minus, minuend, subtrahend, difference

Strategies	Concrete	Pictorial	Abstract
Making 10 Use known number facts and partitioning to support subtraction to the nearest ten before subtracting further. Supports mental subtraction of larger numbers.	 $14 - 5 = 9$ Make 14 on ten frame or with different colours cubes/counters to represent 1 ten and 4 ones. Take away the 4 first to reduce to 10, then take away 1.	Important to discuss what the children have done to reach 10.  $13 - 7 = 6$ 	Children to show they can make 10 by partitioning the subtrahend. $14 - 5 = 9$ $14 - 4 = 10$ 4 $10 - 1 = 9$
Partitioning to subtract Use a deep understanding of place value and partitioning to support subtraction. When not regrouping partitioning should be developed as a mental strategy. <i>Subtract ones then tens to prepare for column subtraction.</i>			$34 - 13 =$ $4 - 3 = 1$ $30 - 10 = 20$ $20 + 1 = 21$
Column method (without regrouping) Use column method and base 10 to subtract without regrouping / exchanging.	48-7 	Represent base 10 pictorially, crossing out as subtracting. 	Subtract ones and then tens. 

Progression in Calculations – SUBTRACTION

Key vocabulary: less than, fewer, take away, subtract, minus, minuend, subtrahend, difference

Strategies	Concrete	Pictorial	Abstract
Column method (with regrouping) Use column method and base 10 to subtract with regrouping / exchanging.	41-25	Represent base 10 pictorially remembering to show the exchange and crossing out as subtracting. <i>Coloured pencil supports understanding.</i>	Children need to understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$. Subtract ones then tens after exchanging. <i>Coloured pencil supports understanding.</i>

Progression in Calculations – MULTIPLICATION

Nursery and Foundation Stage 2

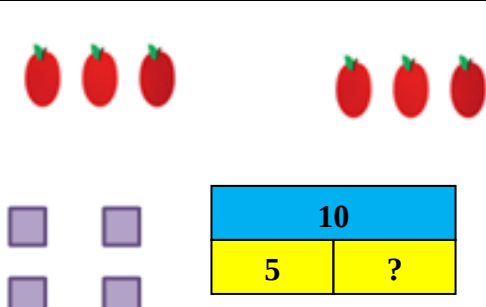
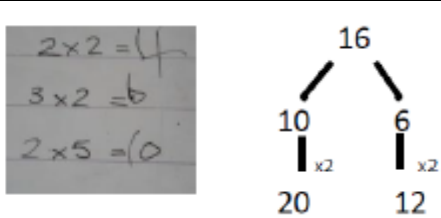
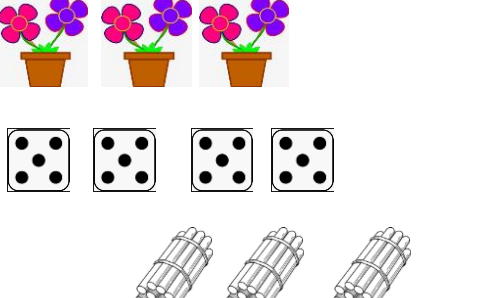
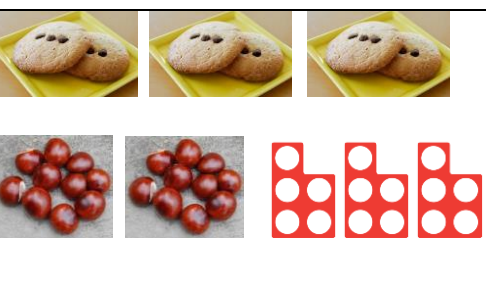
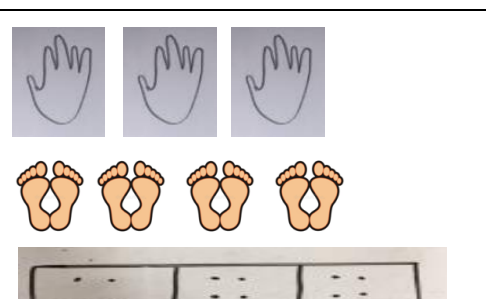
By the end of Foundation Stage 2, children are expected to be able to double some of the numbers to 10.

Before doubling can be introduced, children should have a secure knowledge of counting, number facts and addition.

Children are introduced to the concept of doubling through practical activities, counting songs, games, stories and real-life examples.

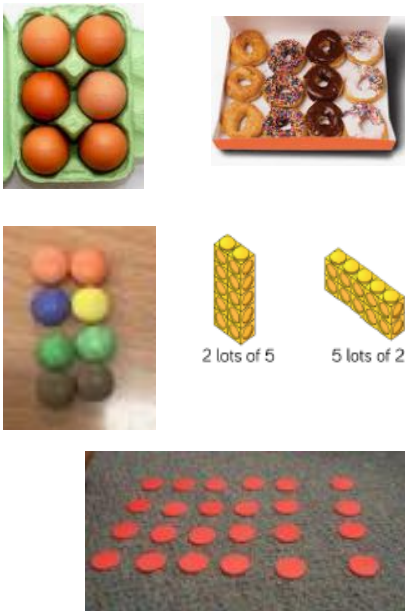
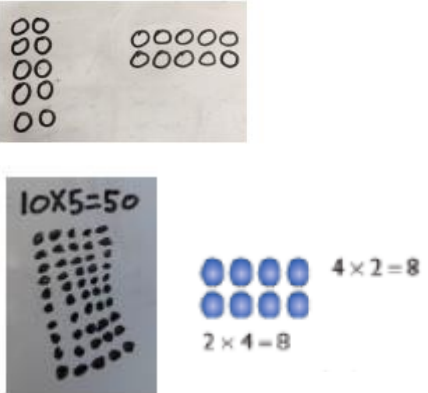
Progression in Calculations – MULTIPLICATION

Key vocabulary: double, equal groups of, groups of, lots of, multiplied by, multiplicand, multiplier, product

Strategies	Concrete	Pictorial	Abstract
Doubling Develop a practical understanding then fluent recall of doubles. Relate to the 2 times table.			 If I can see 10 wheels, how many bikes are there? How do you know?
Counting in multiples Develop an understanding of equal 'groups of' by using skip counting. Increase fluency with frequent practice.			Write skip counting sequences in numbers. Missing number sequences. 0 2 4 _ _ 10 12 14 16 _ If I count in multiples of 5, will I land on 37? How do you know?
Repeated grouping / repeated addition Use skip counting to find the totals.			Write repeated addition / multiplication statements to match objects / images 3 groups of 5 is 15 $5 + 5 + 5 = 15$ $3 \times 5 = 15$

Progression in Calculations – MULTIPLICATION

Key vocabulary: double, equal groups of, groups of, lots of, multiplied by, multiplicand, multiplier, product

Strategies	Concrete	Pictorial	Abstract
Arrays Use arrays to find multiplication and repeated addition equations, develop understanding of commutativity and support written methods.			2 children go to the park to hunt for pine cones. They find 5 each, how many do they find altogether? $5 + 5 = 10$ $2 \times 5 = 10$

Progression in Calculations – DIVISION

Nursery and Foundation Stage 2

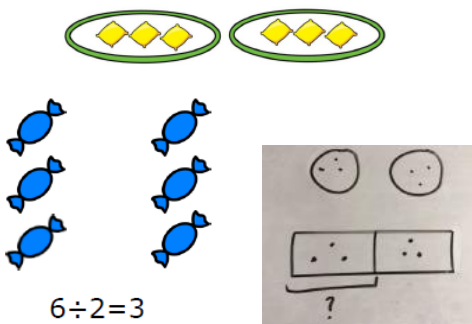
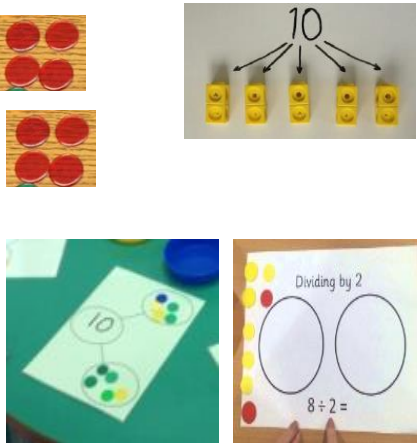
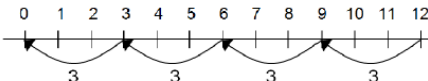
By the end of Foundation Stage 2, children are expected to be able to show how amounts can be distributed equally in numbers to 10.

Before division can be introduced, children should have a secure knowledge of counting, number facts, addition and subtraction.

Children are introduced to the concept of sharing, including halving, through practical activities, counting songs, games, stories and real-life examples.

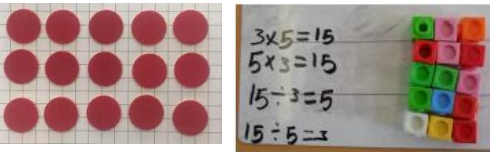
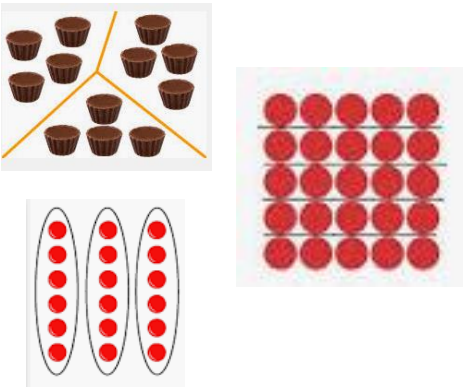
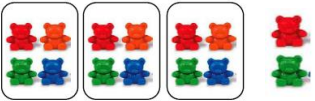
Progression in Calculations – DIVISION

Key vocabulary: share, group, half, divide, divided by, dividend, divisor, quotient

Strategies	Concrete	Pictorial	Abstract
Sharing Show division as sharing. Start with sharing into 2 equal groups and relate to halving.		 $6 \div 2 = 3$	Share 6 apples between 2 people.  $6 \div 2 = 3$ Encourage use of times tables facts to support relationship with division. $2 \times 3 = 6$ therefore $6 \div 2 = 3$
Division as grouping Show division as grouping into equal groups. Use to demonstrate the inverse relationship with multiplication.		$15 \div 3 =$  Show jumps in groups. The number of jumps is the same as the number of groups. 	Divide 35 into 5 groups. $35 \div 5 = 7$ $7 \times 5 = 35$ Word problems e.g. Max is filling party bags with sweets. He has 20 sweets. He decides to put 5 in every bag. How many bags does he fill?

Progression in Calculations – DIVISION

Key vocabulary: share, group, half, divide, divided by, dividend, divisor, quotient

Strategies	Concrete	Pictorial	Abstract
Division with arrays Use arrays to find division equations and develop understanding that division is not commutative.			Create fact families finding 4 related expressions. $7 \times 5 = 35$ $5 \times 7 = 35$ $35 \div 5 = 7$ $35 \div 7 = 5$
Division with a remainder Link to prior learning of odd and even and equality within multiples to explore how numbers can and cannot be divided into whole numbers.	 	 	Complete written division calculations and show remainder as 'r'. $13 \div 5 = 2 \text{ r } 3$